

1 If a single dice is rolled, determine the probability of each of the following events:

- Obtaining the number 5.
- Obtaining an even number.
- Obtaining a number *greater* than 2.

2 If two dice are rolled, determine the probability of each of the following events:

- Obtaining a sum of 2.
- Obtaining a sum of 7.
- Obtaining an even sum.

3 The thief of Bagdad has been placed in a prison which has three doors. One of the doors leads him on a one-day trip, after which he is dumped on his head (which destroys his memory as to which door he chose). Another door is similar except he takes a three-day trip before being dumped on his head. The third door leads to freedom. Assume he chooses a door immediately and with probability  $1/3$  when he has a chance. Find his expected number of days to freedom. (Hint: use conditional expectation.)

4  $R$  is a random variable that is equally likely to be any value between 80 and 100.

- Find  $P(90 \leq R \leq 95)$ .
- Find  $P(90 \leq R \leq 95 \mid 85 \leq R \leq 95)$ .

5 The joint density function of two random variables is

$$f_{X,Y}(x,y) = e^{-x}e^{-y}, \quad x \geq 0, y \geq 0, \\ = 0, \quad \text{elsewhere.}$$

- Find the marginal density function of  $X$ . (Give the answer for all values of the random variable.)
- Find the marginal distribution function of  $Y$ . (Give the answer for all values of the random variable.)

6 Compute the mean and variance of the random variable  $X$  when

a  $p_X(0) = \frac{1}{3}, p_X(2) = \frac{2}{3}$

b  $p_X(x) = \frac{1}{x}, \quad x = 6, 3, 2,$   
 $= 0, \quad \text{elsewhere.}$

c  $p_X(x) = \binom{6}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{6-x}, \quad x = 0, 1, 2, \dots, 6,$   
 $= 0, \quad \text{elsewhere.}$

d  $f_X(x) = 2x, \quad 0 \leq x \leq 1,$   
 $= 0, \quad \text{elsewhere.}$

e  $f_X(x) = x, \quad 0 \leq x \leq 1,$   
 $= 2 - x, \quad 1 \leq x \leq 2,$   
 $= 0, \quad \text{elsewhere.}$

f  $F_X(x) = 0, \quad x < 0,$   
 $= x^{1/2}, \quad 0 \leq x \leq 1,$   
 $= 1, \quad x > 1.$

g  $F_X(x) = 0, \quad x < 0,$   
 $= 1 - e^{-x/2}, \quad x \geq 0.$

7 If  $f_{X,Y}$  is uniform in the region shown in Figure P5-1 that is,

$$f_{X,Y}(x,y) = \frac{1}{2}, \quad 0 \leq x \leq 1, 0 \leq y \leq 1, \\ = \frac{1}{2}, \quad -1 \leq x \leq 0, -1 \leq y \leq 0, \\ = 0, \quad \text{elsewhere,}$$

find  $E[XY]$  and  $\rho$ .

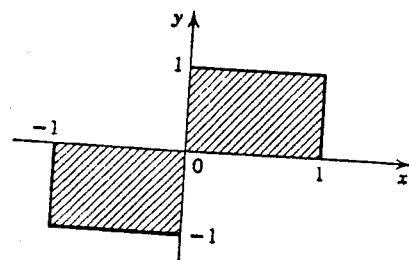


Figure P5-1