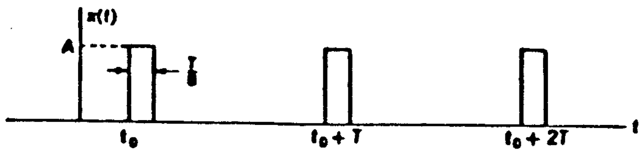
1 . Consider a random process having *periodic* sample functions as shown, in which t_0 is a random variable that is uniformly distributed between 0 and T. Both A and T are constants.



a. Classify this process according to the four pairs of descriptors.
b. Write a probability density function, $p(x)$, for this process.
c. Calculate the mean value, $\bar{x}$, as an ensemble average.
d. Calculate the variance, σ^2, as an ensemble average.
e. Repeat parts c and d using time averages.

2 Sample functions from two statistically independent, stationary random processes $\{x(t)\}$ and $\{y(t)\}$, are used to form two new random processes having sample functions of the form

$$z(t) = x(t) - y(t)$$
$$w(t) = x(t) + y(t)$$

Find the following correlation functions in terms of the autocorrelation functions of $\{x(t)\}$ and $\{y(t)\}$:

a. $R_z(\tau)$
b. $R_w(\tau)$
c. $R_{zw}(\tau)$
d. $R_{wz}(\tau)$

3 .. A stationary random process has an autocorrelation function of the form

$$R_x(\tau) = 25\epsilon^{-|\tau|} + 100$$

a. What is the mean-square value of the process?
b. What is the mean value of the process?
c. What is the variance of the process?

4 Find the mean and autocorrelation function of $X(t) = A\cos(\omega t + \phi)$.

(a) When ω and ϕ are fixed and A is a random variable uniformly distributed between 1 and 3.
(b) When ω is fixed and A and ϕ are independent random variables, A normally distributed with a mean 0 and variance σ^2 and ϕ is uniformly distributed between $-\pi$ and π.
(c) A and ϕ are fixed and ω has a density

$$f_\omega(\lambda) = e^{-\lambda}, \quad \lambda > 0,$$
$$= 0, \quad \lambda < 0.$$