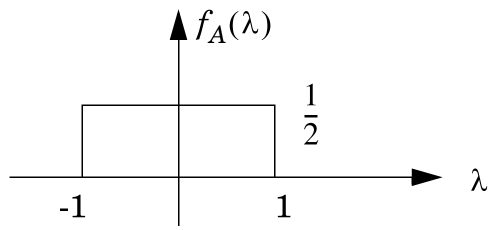
Homework #4

1. Consider the following random process:

$$x(t) = At + 2,$$

where A is a random variable with the following density function



i. Find the mean of $x(t)$.

ii. Find the variance of $x(t)$.

iii. Find the autocorrelation function of $x(t)$.

iv. Sketch one member of the ensemble.

v. Is the process stationary? Explain.

2. We have two independent random variables, x_1 and x_2. We are given that

$$\sigma_{x_1}^2 = 1, \sigma_{x_2}^2 = 2, \mu_{x_1} = 2, \mu_{x_2} = 1.$$

Two other random variables, z_1 and z_2 are defined as:

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ -2x_1 + x_2 \end{bmatrix}$$

i. Compute the mean value of $\mathbf{z}$.

ii. Compute the covariance matrix of $\mathbf{z}$.

iii. If μ_{x_1} is increased from 2 to 4, how will the covariance matrix of $\mathbf{z}$ change?

iv. Based on the information you have, is it possible for the random variables, z_1 and z_2 to be independent. Be very specific and explain your answer.

3. We have a discrete-time (integer t) random process $x(t)$ defined by

- $x(0)$ is normally distributed, with mean zero and variance 1.
- At time step t, $x(t) = x(t-1) + e(t)$, where $e(t)$ is normally distributed with mean zero and variance 1. Each $e(t)$ is independent, and they are all independent of $x(0)$.

i. Find the mean of the process, $E[x(t)]$, for arbitrary t. (Hint: first find $x(t)$ as a function of the $e(t)$'s.)

ii. Find the variance of the process.

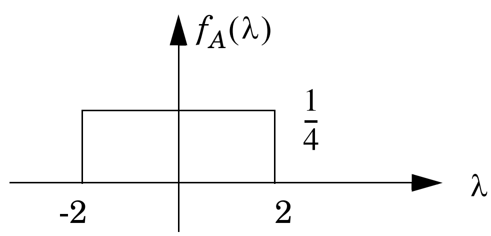
iii. Is the process stationary?

iv. Find the autocorrelation function $R_x(i,j) = E[x(i)x(j)]$.

4. Consider the following random process:

$$x(t) = A\sin(t),$$

where A is a random variable with the following density function



i. Sketch one member of the ensemble.

ii. Find the mean of $x(t)$.

iii. Find the variance of $x(t)$.

iv. Find the autocorrelation function of $x(t)$.

v. Is the process wide sense stationary? Explain.