Homework 3723

For each of the following cases, perform the following steps:

a) By inspection, find the transfer function $G(s)=\dfrac{N(s)}{D(s)}$, where $N(s)$ and $D(s)$ are polynomials in s. $D(s)$ is the characteristic polynomial.

b) Find the system poles (i.e., transfer function poles, roots of the characteristic polynomial).

c) Write out as explicit an expression for $y(t)$ as possible, without any further calculations.

d) Solve for $Y(s)$ in the form

$$Y(s)=\frac{I(s)}{D(s)}+G(s)U(s)$$

where $I(s)$ is a polynomial whose coefficients involve initial conditions, $D(s)$ and $G(s)$ are as defined above, and $U(s)$ is the Laplace transform of the input.

e) Find a common denominator and combine the two terms in $Y(s)$:

$$Y(s)=\frac{I(s)D_u(s)+N(s)N_u(s)}{D(s)D_u(s)}, \text{ where } U(s)=\frac{N_u(s)}{D_u(s)}$$

f) Perform a partial fraction expansion of $Y(s)$, and find $y(t)$. Compare with your answer to part c).

g) Check $y(0)$

1) $\ddot{y}+2\dot{y}+5y=3\dot{u}+4u$, $y(0)=1, \dot{y}(0)=0, u(t)=e^{-5t}1(t)$
2) $\ddot{y}+3\dot{y}+2y=4u$, $y(0)=3, \dot{y}(0)=2, u(t)=e^{-5t}\sin(4t)1(t)$
3) $\ddot{y}+6\dot{y}+13y=3\dot{u}+11u$, $y(0)=0, \dot{y}(0)=0, u(t)=e^{-2t}1(t)$
4) $\dddot{y}+2\ddot{y}+5\dot{y}=10\dot{u}+10u$, $y(0)=0$, $\dot{y}(0)=0$, $\ddot{y}(0)=0$, $u(t)=e^{-2t}1(t)$
5) $\dot{y}+2y=u$, $y(0)=0$, $u(t)=2\sin(2t)1(t)$
6) $\ddot{y}+8\dot{y}+20y=3\dot{u}+4u$, $y(0)=1, \dot{y}(0)=2$, $u(t)=5\exp(-8t)1(t)$
7) $\ddot{y}+6\dot{y}+34y=4\dot{u}+7u$, $y(0)=2, \dot{y}(0)=3, u(t)=8\sin(2t)1(t)$
8) $\ddot{y}+4\dot{y}+13y=54u$, $y(0)=5, \dot{y}(0)=-16, u(t)=e^{-5t}1(t)$
9) $\ddot{y}+6\dot{y}+10y=\dot{u}+2u$, $y(0)=1, \dot{y}(0)=0, u(t)=2e^{-3t}1(t)$
10 $\ddot{y}+3\dot{y}+2y=\dot{u}+4u$, $y(0)=0, \dot{y}(0)=0, u(t)=2\sin(3t)1(t)$